

Sr. No.	Core Areas
1	Foundation of Mathematics
2.	Elementary Mathematics
3.	Calculus
4.	Algebra
5.	Complex Analysis
6.	Differential Equation
7.	Analytic Geometry
8.	Vectors
9.	Number Theory
10.	Real Analysis
11.	Functional Analysis
12.	Computational Mathematics
13	Mathematical Statistics & Probability

**Foundation of mathematics:**

Basic notions, set operations, extended-set operations, indexed family of sets, countable and uncountable sets, relations, cardinality, equivalence relations, congruence, partitions, partial order, representation of relations, mathematical induction. Axiom of choice, Zorn's lemma, continuum hypothesis, propositional calculus, predicate calculus, law of excluded middle

**Elementary Mathematics**

Sets, types of sets, Basic operations on sets, Venn diagrams, verification of commutative, associative, distributive and De Morgan's laws through sets and Venn diagrams, whole numbers, integers, factors and multipliers, fractions, decimals, percentages, unitary method, BODMAS rule, ratio and proportion, financial arithmetic, algebraic operations, linear equations, distance, time and temperature, line segments, types of angles, types of triangles, types of quadrilaterals, perimeter and area, average, graphs (block, column, bar and pie)

Rational numbers, real numbers, number systems with bases 2 and 10 and their conversions, exponents, square root of positive numbers, cubes and cube roots, HCF and LCM (using division and prime factorization) direct and inverse relations, taxes, profit, loss, discount and markup, compound proportion, income tax, Zakat and Ushr, operations with polynomials, algebraic identities involving  $(x+a)(x-a)$ ,  $(a+b)^2$ ,  $(a-b)^2$  and  $a^2-b^2$ , factorization of algebraic expressions, simultaneous equations, solution by comparison, substitution, elimination and cross-multiplication, properties of angles, congruent and similar figures, congruent triangles, circumference and area of a circle, surface area and volume of sphere and cylinder, frequency distribution

**CALCULUS:**

Functions, Inverse functions, Range and domain, Properties of functions, Composite functions, Even and odd functions, Shift of graphs, Types of functions, parametric equations, Limits, Infinite limit, vertical and horizontal asymptotes, Continuity, Piece wise continuity, and uniform continuity, infinite discontinuity, limit at infinity

Differentiability, Differentiation, Techniques of differentiation, Applications of differentiation (monotonicity of functions, critical points and points of inflections, extrema both relative and absolute, Optimization problems) Mean value theorem, Taylor's theorem, Taylor series, Curve sketching, Implicit differentiation

Integral, definite and indefinite integrals, proper and improper integrals, Fundamental theorems of calculus, Techniques of integration, Applications of definite integrals (area, arc length)

**ALGEBRA:**

Basic axioms of a group, abelian groups, center of a group, subgroups, cyclic groups, cosets and quotient sets, Lagrange's theorem, even and odd permutations, cycles, length of a cycle, transpositions, symmetric and non-symmetric groups, alternating groups, normalizers and centralizers of a subset of a group, congruency classes of a group, normal subgroup, quotient

groups, conjugacy, homomorphism and isomorphism between groups, homomorphism and isomorphism theorems, group of automorphisms, finite p-groups, internal and external direct products, group action on sets, orbits, 1st, 2nd and 3rd Sylow theorems

Matrix and its types, determinants and its properties, inverse of a matrix, row and column operations, echelon and reduced echelon form, rank of a matrix, consistent and inconsistent systems (conditions for the existence of zero, one or infinite solutions), solution of non-homogenous equations (Gauss-elimination method, Gauss-Jordan method, inverse method, Cramer's rule), solution of homogenous equations and eigenvalue problems, groups and subgroups of matrices

Vector spaces, subspaces, linear span of subset of a vector space, bases and dimension of a vector space, sums and direct sums of subspaces of a finite dimensional vector space, dimension theorem, linear transformation, null space, image space of a linear transformation, matrix of a linear transformation, rank and nullity of a linear transformation, relation between rank, nullity and dimension of domain of a linear transformation, orthogonal transformation, change of basis, inner-product spaces, projection of a vector on another vector, norm of a vector, Cauchy-Schwartz inequality,

### **COMPLEX ANALYSIS:**

The algebra and the geometry of complex numbers. Complex valued functions and their types, elementary functions, complex exponents, limit continuity and differentiability

Analytic functions, Cauchy-Riemann equations, harmonic functions, contours and contour integrals, the Cauchy-Goursat Theorem, the Cauchy integral formulae, the Morera Theorem, maximum modulus principle, the Liouville theorem, fundamental theorem of algebra

Convergence of sequences and series, the Taylor series, power series representation of analytic functions, the Laurent series, branch point, zeros of analytic functions, residues and poles, the residue theorem, evaluation of improper integrals involving trigonometric functions

### **DIFFERENTIAL EQUATIONS:**

Formation and solution of first-order-differential equations, formation and solution of second and higher-order-linear-differential equations; differential equations with variable coefficients, Sturm-Liouville (S-L) system, initial and boundary-value problems, series solution, the Frobenius method, solution of the Bessel, the hypergeometric, the Legendre and the Hermite equations, properties of the Bessel functions

First-order-partial-differential equations, classification of second-order partial-differential equations, canonical form for second-order equations; wave, heat and the Laplace equations in Cartesian, cylindrical and spherical-polar coordinates; solution of partial-differential equation by the methods of: separation of variables, the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations

### **Analytic GEOMETRY:**

Cartesian-coordinate system, slope of a straight line, equation of a straight line, parallel and perpendicular lines, various forms of equation of a line, intersection of two lines, angle between two lines, distance between two points, distance between a point and a line, equation of a circle, circles formed under various conditions, intersection of lines and circles

Conic section, the general-second-degree equation. (circle, parabola, ellipse and hyperbola) and their properties

### **VECTORS:**

Vectors scalar, products, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coordinate systems and their bases, the spherical-polar- and the cylindrical-coordinate systems

Tensors of zero, first, second and higher orders, algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, tensors in modeling anisotropic systems, physical tensors (moment of inertia, index of refraction), diagonalization of inertia tensor as aligning coordinate frame with natural symmetries of the system

### **NUMBER THEORY:**

Divisibility, euclidean algorithm, GCD and LCM of 2 integers, properties of prime numbers, fundamental theorem of arithmetic, congruence relation, residue system, Euler's phi-function, solution of system of linear congruence's, Chinese remainder theorem, Fermat little theorem, Wilson theorem, primitive roots and indices; integers belonging to a given exponent (mod p), primitive roots of prime and composite moduli, indices, solutions of congruence's using indices

### **REAL ANALYSIS:**

Ordered sets, supremum and infimum, accumulation point, completeness properties of the real numbers, limits and continuity, properties of continuous functions on closed bounded intervals, derivatives, the mean value theorem, sequences and series of functions, power series, point-wise and uniform convergence, functions of several variables, open and closed sets and convergence of sequences in  $\mathbb{R}^n$ ; compactness, limits and continuity in several variables

Series of numbers and their convergence, Darboux upper and lower sums and integrals, Darboux integrability, Riemann sums and Riemann integrals, Riemann integration, change of order of variables of integration, Riemann-Stieltjes integration, functions of bounded variation

### **FUNCTIONAL ANALYSIS:**

Concept and properties of metric space, Cauchy sequence, completeness and convergence

Linear spaces, normed spaces, Banach spaces, Bounded and continuous linear operators and functionals, dual spaces, finite-dimensional spaces, F. Riesz lemma. The Hahn Banach theorem for complex spaces, the HB theorem for normed spaces, the open mapping theorem, the Banach-fixed-point theorem

Inner-product space, Hilbert space, orthogonal and orthonormal sets, orthogonal complements, Gram-Schmidt orthogonalization process, Reiz-representation theorem

### **COMPUTATIONAL MATHEMATICS:**

Computer arithmetic, approximations and errors; methods for the solution of nonlinear equations and their convergence: bisection method, regula falsi method, fixed point iteration method, Newton-Raphson method, secant method; error analysis for iterative methods. Interpolation and polynomial approximation: Lagrange interpolation

Numerical solution of systems of algebraic linear equations, Gauss-elimination method, Gauss-Jordan method; matrix inversion; LU-factorization; Doolittle's, Cholesky's methods; Gauss-Seidel and Jacobi methods

Newton's divided difference, forward-difference and backward-difference formulae, Hermite interpolation, numerical differentiation, integration and their error estimates, rectangular rule, trapezoidal rule, Simpson's one-third and three-eighths rules

### **MATHEMATICAL STATISTICS AND PROBABILITY:**

Measure of central tendency (mean, median, mode, geometric and harmonic means) Distributions (binomial, Poisson, normal, hypergeometric), estimation of parameters, estimation of mean, variance, confidence intervals, hypothesis testing and decision making, types of errors in tests, quality control, control charts for mean, standard deviation, variance, range, goodness of fit, chi-square test, regression analysis, method of least squares, Probability