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On Reverse Super Edge Magic Total Labeling of Subdivided Trees

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Abstract

A reverse edge magic total labeling of a graph G is a one-to-one map $\lambda : V(G) \cup E(G) \rightarrow \{1,2,\dots,|V(G) \cup E(G)|\} = [1,|V(G) \cup E(G)|]$ with the property that there is an integer constant k such that $\{\lambda(xy) - (\lambda(x) + \lambda(y))/xy \in V(G)\} = k$. If $(V(G)) = [1,|V(G)|]$ then the reverse edge magic labeling is called reverse super edge-magic labeling. In this paper we will formulate the reverse edge magic labeling of two subclasses of trees.

***Keywords*:** star, subdivision of star, reverse super edge magic labeling

Introduction

All graphs in this paper are finite, undirected, simple and planar. The graph G has the vertex set V(G) and the edge set E(G). A (v,e) -graph G is a graph such that |V(G) |= v and | E(G) |= e. A general reference for graph-theoretic ideas can be seen in [1,2]. A labeling (or valuation) of a graph is a map that converts graph elements into numbers (usually to positive or non-negative integers). In this paper, the domain will be the set of all vertices and edges and such a labeling is known as total labeling. Some labeling alternatively uses either the vertex-set or the edge-set and we shall address them as vertex-labeling or edge-labeling, respectively.

Definition 1.1. A reverse edge magic total labeling of a graph G is a one-to-one map $\lambda : V(G) \cup E(G) \rightarrow \{1,2,\dots,|V(G) \cup E(G)|\} = [1,|V(G) \cup E(G)|]$ with the property that there is an integer constant k such that $\{\lambda(xy) - (\lambda(x) + \lambda(y))/xy \in V(G)\} = k$.

Definition 1.2. λ is called the reverse super edge magic total labeling and G is known as a reverse super edge magic total graph. Enomoto et al. [3] proposed the following conjecture,

Conjecture 1.1. *Every tree admits a super edge magic total labeling.*

In favor of this conjecture, many authors have considered a super edge magic total labeling for different classes of trees. For detailed studies the reader can see [4–14].

Definition 1.3. Let $n_i \geq 1, 1 \leq i \leq r$ and $r \geq 2$. A subdivided star $Sb(n_1,n_2,\dots,n_r)$ is a tree obtained by inserting $n_i - 1$ nodes to each of the ith edge of the star $K_{1,r}$. Let us define the set of nodes and edges as follows,

$V(G) = \{c\} \cup \{{x_i}^{l_i} | 1 \leq i \leq r;\ 1 \leq l \leq n_i\}$ and $E(G) = \{c{x_i}^{l} | 1 \leq i \leq r\} \cup \{{x_i}^{l_i}{x_i}^{l_i+1} | 1 \leq i \leq r;\ 1 \leq l \leq n_i - 1\}$.

However, the investigation of different results related to a reverse super edge magic total labeling of the subdivided star $Sb(n_1,n_2,\dots,n_r)$ for $n_1 \neq n_2, \dots, \neq n_r$ is still an open problem. In this paper, we will formulate a reverse super edge magic total labeling of the subclasses of subdivided stars denoted by $Sb(mn, mn, mn, 2mn, n_6, n_r, \dots, n_r)$ and $Sb(mn, mn. 2n, 2n + 2, 4n + 3, n_6, \dots, n_r)$ under certain conditions.

Let us consider the following Lemma which we will use frequently in the main theorems.

Lemma 1.1. A graph with vertices v and e edges is reverse super edge magic total labeling if and only if there exists a bijective function $\lambda: V(G) \rightarrow [1, v]$ such that the set consists of consecutive integers. In such a case, λ extends to a reverse super edge magic total labeling of G with magic constant $k = 2v - s - 1$, where $s = \max(S)$.

2. Main Results

In this section, we will prove the main results related to a reverse super edge magic total labeling of the more generalized subclasses of subdivided trees.

Theorem 2.1. The graph $G \cong Sb(n, n, n, n, 2n, n_6, \dots, n_r)$ admits the reverse edge magic total labeling for any odd $n \geq 3$, $r \geq 6$, $n_p = 2^{p-4}n - 2p + 11$ and $6 \leq p \leq r$

Proof Let $v = |V(G)|$ and $e = |E(G)|$ then

$v = 6n + 1 + \sum_{t=6}^{r}[2^{t-4}n - 2t]$ and $e = v - 1$
Let us define $\lambda : V(G) \rightarrow [1, v]$ as follows,

$$\lambda(c) = (4n + 2) + \sum_{t=6}^{r}[2^{t-5}n - t + 6]$$

For odd $1 \leq l \leq n_i$, where $1 \leq i \leq 5$ and for $1 \leq i \leq r$:

$$\lambda(w) = \begin{cases} \frac{l_1 + 1}{2}, & if\ w = {x_1}^{l_1} \\ n + 1 - \frac{l_2 - 1}{2}, & if\ w = {x_2}^{l_2} \\ (n + 2) + \frac{l_3 - 1}{2}, & if\ w = {x_3}^{l_3} \\ 2(n + 1) + \frac{l_4 - 1}{2}, & if\ w = {x_4}^{l_4} \\ (3n + 2) - \frac{l_5 - 1}{2}, & if\ w = {x_5}^{l_5} \end{cases}$$

$\lambda({x_i}^{l_i}) = (3n + 2) + \sum_{t=6}^{i}[2^{t-5}n - t + 6] - \frac{l_i - 1}{2}$ Accordingly.
For even $1 \leq l \leq n_i$ and $\gamma = (3n + 2) + \sum_{t=6}^{r}[2^{t-6}n + 1]$, where $1 \leq i \leq 5$ and for $6 \leq i \leq r$:

$$\lambda(w)=\begin{cases}(\gamma+1)\frac{l_1-2}{2}, & if\ w={x_1}^{l_1}\\ (\gamma+n-1)-\frac{l_2-2}{2}, & if\ w={x_2}^{l_2}\\ (\gamma+n+1)+\frac{l_3-2}{2}, & if\ w={x_3}^{l_3}\\ (\gamma+2n-1)+\frac{l_4-2}{2}, & if\ w={x_4}^{l_4}\\ (\gamma+3n-1)-\frac{l_5-2}{2}, & if\ w={x_5}^{l_5}\end{cases}$$

$\lambda({x_i}^{l_i})=(\gamma+3n-1)+\sum_{t=6}^{i}\ [2^{t-5}(3n)-2t+11]-\frac{l_i-2}{2}$ Accordingly.
By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S=[\gamma+2,\gamma+1+e]$. Therefore, by Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant $k=(3n-2)+\sum_{t=6}^{i}[2^{t-6}(3n)-2t+10]$.

Theorem 2.2. The graph $G\cong T(3n,3n,3n,3n,6n,n_6,\dots,n_r)$ admits the reverse edge magic total labeling with $k=2v-s-1$ for any odd $n\geq 3$, $r\geq 6$, $n_p=2^{t-6}(3n)-2t+11$ and $6\leq p\leq r$

Proof

Let Let $v=|V(G)|$ and $e=|E(G)|$ then

$v=(18n+1)+\sum_{t=6}^{r}[2^{t-4}(3n)-2t+11]$ and $e=v-1$
Let us define $\lambda: V(G)\to[1,v]$ as follows,

$$\lambda(c)=(12n+2)+\sum_{t=6}^{r}[2^{t-5}(3n)-t+6]$$

For odd $1\leq l\leq n_i$, where $1\leq i\leq 5$ and for $1\leq i\leq r$:

$$\lambda(w)=\begin{cases}\frac{l_1+1}{2}, & if\ w={x_1}^{l_1}\\ 3n+1-\frac{l_2-1}{2}, & if\ w={x_2}^{l_2}\\ (3n+2)+\frac{l_3-1}{2}, & if\ w={x_3}^{l_3}\\ (6n+2)-\frac{l_4-1}{2}, & if\ w={x_4}^{l_4}\\ (9n+2)-\frac{l_5-1}{2}, & if\ w={x_5}^{l_5}\end{cases}$$

$\lambda({x_i}^{l_i})=(9n+2)+\sum_{t=6}^{i}[2^{t-5}(3n)-t+6]-\frac{l_i-1}{2}$ Accordingly.
For even $1\leq l\leq n_i$ and $\gamma=(9n+2)+\sum_{t=6}^{r}2^{t-5}(3n)-t+6$, where $1\leq i\leq 5$ and for $6\leq i\leq r$:

$$\lambda(w)=\begin{cases}(\gamma+1)\frac{l_1-2}{2}, & if\ w={x_1}^{l_1}\\ (\gamma+3n-1)-\frac{l_2-2}{2}, & if\ w={x_2}^{l_2}\\ (\gamma+3n+1)+\frac{l_3-2}{2}, & if\ w={x_3}^{l_3}\\ (\gamma+6n-1)-\frac{l_4-2}{2}, & if\ w={x_4}^{l_4}\\ (\gamma+9n-1)-\frac{l_5-2}{2}, & if\ w={x_5}^{l_5}\end{cases}$$

$\lambda({x_i}^{l_i})=(\gamma+9n-1)+\sum_{t=6}^{i}\ [2^{t-5}(3n)-2t+11]-\frac{l_i-2}{2}$ Accordingly.

By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S=[\gamma+2,\gamma+1+e]$. Therefore, by Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant$k=(9n-2)+\sum_{t=6}^{i}[2^{t-6}(3n)-t+5]$.

Theorem 2.3 The graph $G\cong Sb(mn,mn,mn,2mn,n_6,\dots,n_r)$ admits the reverse edge magic total labeling with $k=2v-s-1$ for any odd $n\geq 3$, $r\geq 6$, $n_p=2^{p-4}kn-2p+11$ and $6\leq p\leq r$

Proof

Let Let $v=|V(G)\ |$and $e=|E(G)|$ then

$v=(6kn+1)+\sum_{t=6}^{r}[2^{m-4}kn-2m+11]$ and $e=v-1$

Let us define $\lambda: V(G)\to[1,v]$ as follows,

$$\lambda(c)=(4kn+2)+\sum_{t=6}^{r}[2^{m-5}kn-m+6]$$

For odd $1\leq l\leq n_i$, where $1\leq i\leq 5$ and for $1\leq i\leq r$:

$$\lambda(w)=\begin{cases}\frac{l_1+1}{2}, & if\ w={x_1}^{l_1}\\ mn+1-\frac{l_2-1}{2}, & if\ w={x_2}^{l_2}\\ (mn+2)+\frac{l_3-1}{2}, & if\ w={x_3}^{l_3}\\ 2(mn+2)-\frac{l_4-1}{2}, & if\ w={x_4}^{l_4}\\ (3mn+2)-\frac{l_5-1}{2}, & if\ w={x_5}^{l_5}\end{cases}$$

$\lambda({x_i}^{l_i})=(3mn+2)+\sum_{t=6}^{i}[2^{t-5}mn-m+6]-\frac{l_i-1}{2}$ Accordingly.

For even $1 \leq l \leq n_i$ and $\gamma = (3kn + 2) + \sum_{t=6}^{r}[2^{t-6}2kn - (m - 6)]$, where $1 \leq i \leq 5$ and for $6 \leq i \leq r$:

$$\lambda(w) = \begin{cases} (\gamma + 1)\dfrac{l_1 - 2}{2}, & if\ w = x_1{}^{l_1} \\ (\gamma + mn - 1) - \dfrac{l_2 - 2}{2}, & if\ w = x_2{}^{l_2} \\ (\gamma + mn + 1) + \dfrac{l_3 - 2}{2}, & if\ w = x_3{}^{l_3} \\ (\gamma + 2mn - 1) - \dfrac{l_4 - 2}{2}, & if\ w = x_4{}^{l_4} \\ (\gamma + 3mn - 1) - \dfrac{l_5 - 2}{2}, & if\ w = x_5{}^{l_5} \end{cases}$$

$\lambda(x_i{}^{l_i}) = (\gamma + 3mn - 1) + \sum_{t=6}^{i}\ [2^{t-5}(mn) - 2m + 11] - \frac{l_i - 2}{2}$ Accordingly.

By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S = [\gamma + 2, \gamma + 1 + e]$. Therefore, by Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant$k = (3mn - 2) + \sum_{t=6}^{i}[2^{t-6}(mn) - t + 10]$.

Theorem 2.4 The graph $G \cong Sb(mn, mn. 2n, 2n + 2, 4n + 3, n_6, \dots, n_r)$ admits the reverse edge magic total labeling with$k = 2v - s - 1$, for any odd $n \geq 3, r \geq 6, n_p = 2^{p-5}(4n + 2) + 1$ and $6 \leq p \leq r$

Proof

Let Let $v = |V(G)\ |$and $e = |E(G)|$ then

$v = [(2k + 8)n + 6] + \sum_{t=6}^{r}[2^{m-5}(4n + 2) + 1\ ]$ and $e = v - 1$

Let us define $\lambda: V(G) \rightarrow [1, v]$ as follows,

$$\lambda(c) = [(2k + 4)n + 4] + \sum_{t=6}^{r}[2^{m-6}(4n + 2) + 1\ ]$$

For odd $1 \leq l \leq n_i$, where $1 \leq i \leq 5$ and for $6 \leq i \leq r$:

$$\lambda(w) = \begin{cases} \dfrac{l_1 + 1}{2}, & if\ w = x_1{}^{l_1} \\ mn + 1 - \dfrac{l_2 - 1}{2}, & if\ w = x_2{}^{l_2} \\ (mn + 2) + \dfrac{l_3 - 1}{2}, & if\ w = x_3{}^{l_3} \\ (m + 2)n + 2 - \dfrac{l_4 - 1}{2}, & if\ w = x_4{}^{l_4} \\ (m + 2)n + 4 - \dfrac{l_5 - 1}{2}, & if\ w = x_5{}^{l_5} \end{cases}$$

$\lambda(x_i{}^{l_i}) = [(2k+4)n+4] + \sum_{t=6}^{r}[2^{m-6}(4n+2)+1\,]\,\frac{l_i-1}{2}$ Accordingly.

For even $1 \le l \le n_i$ and $\gamma = (3kn+2) + \sum_{t=6}^{r}[2^{t-6}2kn-(m-6)]$, where $1 \le i \le 5$ and for $6 \le i \le r$:

$$\lambda(w) = \begin{cases} (\gamma+1)\dfrac{l_1-2}{2}, & if\ w = x_1{}^{l_1} \\ (\gamma+mn-1)-\dfrac{l_2-2}{2}, & if\ w = x_2{}^{l_2} \\ (\gamma+mn+1)+\dfrac{l_3-2}{2}, & if\ w = x_3{}^{l_3} \\ (\gamma+(m+2)n-1)-\dfrac{l_4-2}{2}, & if\ w = x_4{}^{l_4} \\ (\gamma+(m+2)n+2)-\dfrac{l_5-2}{2}, & if\ w = x_5{}^{l_5} \end{cases}$$

$\lambda(x_i{}^{l_i}) = [\gamma+(m+4)n+2] + \sum_{t=6}^{i}\,[2^{t-6}(4n+2)+1] - \frac{l_i-2}{2}$ Accordingly.

By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S = [\gamma+2, \gamma+1+e]$. Therefore, by Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant$k = (mn+4) + \sum_{t=6}^{i}[2^{t-6}(4n+2)]$.

3. Conclusion

In this paper, we have proved that the following subclasses of subdivided stars admit reverse super edge magic total labeling,

- For $n \ge 3$ and $k \ge 1$ are odd, $r \ge 6$, $Sb(mn, mn, mn, 2mn, n_6, \dots, n_r)$ with $n_p = 2^{p-4} - 2p + 11$ for $6 \le p \le r$.
- For $n \ge 3$ and $k \ge 1$ are odd, $r \ge 6$, $Sb(mn, mn, 2n, 2n+2, 4n+3, n_6, \dots, n_r)$ with $n_p = 2^{p-5}(4n+2) + 1$ for $6 \le p \le r$.

The problem is still open for the remaining subclasses of subdivided stars with different combinations of m and n.

References

[01] West DB. *An introduction to graph theory*. New Jersey: Prentice-Hall, 1996.

[02] Gallian JA. A dynamic survey of graph labeling. *Electron J Combin.* 2017;17:42–45.

[03] Enomoto H, Lladó, AS, Nakamigawa T, Ringel G. Super edge-magic graphs, *SUT J Math.* 1998;34:105–109.

[04] Baskoro ET, Sudarsana, IW, Cholily YM, How to construct new super edge-magic graphs from some old ones. *J Indones Math Soc (MIHIM).* 2005;11(2):155–162.

[05] Figueroa-Centeno RM, Ichishima R, Muntaner-Batle FA. On super edge-magic graph. *Ars Combinatori.* 2002;64: 81–95.

[06] Javaid M, Hussain M, Ali K, Dar KH. Super edge-magic total labeling on w-trees. *Utilitas Math.* 2011;86:183–191.

[07] Ngurah AAG, Simanjuntak R, Baskoro ET. On (super) edge-magic total labeling of subdivision of $K_{1,3}$. *SUT J Math.* 2007;43:127–136.

[08] Salman ANM, Ngurah, AAG, Izzati N. On super edge-magic total labeling of a subdivision of a Star S_n. *Utilitas Mthematica*. 2010;81:275–284.

[09] Sharief BS. Reverse super edge magic labeling on w-trees. *Int J Comput Eng Res Trends*. 2015;2(11):719–721.

[10] Sharief BS. Madhusudhan RK. Reverse magic strength of festoon trees. *Ital J Pure Appl Math.* 2014;33:191–200.

[11] Raheem AM, Javaid M, Baig AQ. On antimagicness of subdivided stars. *Discuss Math Graph Theory*. 2015;35:663–673.

[12] Raheem A, Baig AQ. Antimagic labeling of the union of subdivided stars, *TWMS J Appl Eng Math.* 2016;6:244–250.

[13] Raheem A, Baig AQ, Javaid M. On (a,d)-EAT labeling of subdivision of $K_{1,r}$. *J Inf Optim Sci*. 2018;39(3):643–650.

[14] Venkata RS, Sharief BS. On reverse super edge-magic labeling of graphs, *Int Rev Pure Appl Math*. 2010;6(1):181–188.