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Generalization of TOPSIS from Soft Set to Fuzzy Soft Sets in Decision Making Problem

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Abstract

As of late, the issue of basic decision making has discovered imperative centrality. It has procured central significance particularly for the issues identified with incorrect environment. The technique for order of preference by similarities to ideal solution (TOPSIS) is the multi criteria choice examining technique utilized for the choice. We apply the generalized result of TOPSIS on soft set to TOPSIS on fuzzy-soft-set.

Keywords: *TOPSIS, Soft-sets (SS), fuzzy-soft-sets (FSS), weighted-normalized-decision-matrix (WNDM) and fuzzy-sets (FS).*

Introduction

In real life, we face many problems of decision making in different domains of sciences such as engineering, medical science and economics. To address uncertain situation and treat ambiguous data, Zadeh [1] presented the idea of fuzzy sets in 1965. Introduction of fuzzy set theory revolutionized the entire mathematical sciences. Many researchers contributed in the development of fuzzy sets and its applications in various fields of life.

In 1980 Thomas Saaty [2] introduced Analytical Hierarchy process. These are most useful techniques for dealing difficult decision making and help the decision makers to make accurate decisions. This theory consists of many applications such as, medicine, computer science, control engineering and artificial intelligence etc.

To ease this situation of decision making a methodology was developed by Hwang and Yoon [3] in 1981, named as TOPSIS. But TOPSIS on soft set is useful for discrete situations. In 1999 Molodtsov proposed the theory of soft-sets as a generalization of fuzzy set, which can efficiently handle a number of parameters simultaneously.

The idea of fuzzy-soft-set was first presented by Maji et al. in 2001. In 2002, firstly the technique of fuzzy-soft-set was used in decision making. In 2013, Maji et al. presented the idea of the neutrosophic-soft-set. For decision making many techniques are used such as soft expert set, AHP, interval valued fuzzy soft matrix, TOPSIS etc. These techniques are proved to be very helpful in decision making. M. Saeed, Sana. A. and N. Rubi [4] compared the two different technique **,** fuzzy-soft-expert-set and AHP. The computational cost of Fuzzy-Soft-Expert-Set system is minimum than AHP procedure for the same outcome. They conclude that AHP and fuzzy-soft-expert-set gives the same result.

Zulqarnain. M and M. Saeed [5] have applied fuzzy-soft-set in decision making. The idea of fuzzy-soft-set has been presented earlier [6]. But decision making on fuzzy-soft-set by applying TOPSIS is presented for the first time in this paper. In this paper, we generalized the concept of TOPSIS from soft-set to fuzzy-soft-sets in decision making. In this paper the author present some basic concepts associated to soft-set and fuzzy-soft-set and procedure of TOPSIS. Also develop the methodology of decision making via TOPSIS on fuzzy-soft-set and conclude the results.

2. Preliminaries

In this subcategory, we present the fundamental concepts and outcomes of soft-set-theory**,** which would be beneficial for additional dialogues. Maximum descriptions and outcomes obtainable in this subdivision may be established in [7-9].

Definition1

"Let *S* be a set of parameters and *W* be an initial universe set. Let *W* be the power set denoted by *P (U)* the and $A \subset S$. A pair *(G, A)* is called a soft set over *W*, given by

$$G: A \rightarrow P\ (U)\text{"}$$

Where *G* is a mapping [10]

Fuzzy-Soft-Set in Decision-Making

In this section, we will discuss some elementary description of fuzzy soft set (FSS) and some outcomes**,** which we use in further debate. Most of them are originating in [11,12].

The parameter space *S* may be written as

$$S \supseteq \{ B_1 \cup B_2 ...... \cup B_i \}.$$

Let each parameter set Y_j characterize the jth class of parameters and the elements of B_j represents a particular property set. Here we suppose that these property sets may be observed as fuzzy-sets (FS).

Definition2

"Let P *(V)* represents the set of complete fuzzy-set of *V*. Let $X_i \subset E$. A pair is (H_i, X_i) is called a fuzzy-soft-set over *V*. We may now describe a FSS (G_j, B_j) which illustrates a set of objects having the parameter set [13], where H_i is a mapping given by $H_i : X_i \to P(V)$. [14]

TOPSIS

The basic steps of TOPSIS are described in [15]

Stage1. First determine the normalized-decision-matrix.
The normalized value is denoted by x_{ij} is calculated as follows:

$$x_{ij} = y_{ij}\sqrt{\sum_{i=1}^{n} y_{ij}^2} \text{ i =1, 2,..., n and j = 1, 2, ..., m.}$$

Stage 2. Evaluate the weighted normalized decision-matrix. The weighted normalized value U_{ij} is calculated as follows:

$$U_{ij} = X_{ij} \times W_i,$$
$$i = 1, 2..., n \text{ and } j = 1, 2... m. [16]$$

where W_i is the weight of the i^{th} alternative and $\sum_{i=1}^{n} W_i = 1$.

Stage3. Calculate the best ideal solution (B^*) and worst ideal (B^-) solutions [17, 18].

$$B^* = \{(\max_i u_{ij} \mid i \in D_a), (\min_i u_{ij} \mid i \in D_c)\} = \{u_i^* \mid i = 1,2,...,n\}$$
$$B^- = \{(\min_i u_{ij} \mid i \in D_b), (\max_i U_{ij} \mid i \in D_c)\} = \{u_i^- \mid i = 1,2,...,n\}$$

Stage4. By using the n-dimensional Euclidean space compute the distance. The parting procedures of each substitute from the best-ideal-solution and the worst-ideal-solution, respectively, as given below: [19, 20]

$$D_i^* = \sqrt{\sum_{i=1}^{n} (u_{ij} - u_i^*)^2, i = 1,2,...,n}$$
$$D_i^- = \sqrt{\sum_{i=1}^{n} (u_{ij} - u_j^-)^2, i = 1,2,...,n}$$

Stage5. Compute the relative closeness to the best-ideal-solution. The relative closeness of the alternative with respect X_i to B^* is defined as follows [21]:

$$RC_i^* = \frac{B_i^-}{B_i^* + B_i^-}, i = 1,2,...,n$$

Stage 6. Rank the preference order.

A Decision Making Method on Fuzzy Soft Set

In this section, we deliberate the decision-making method by using TOPSIS on soft-set-theory. The detailed method, of each step, is shown below.

Step1. Defining the problem. Let us assume that

$$DM = \{D_p, p \in I_n\} \; are\ the\ sets\ of\ decision\ makers,$$
$$v = \{v_{i,} i \in I_m\} denote\ the\ set\ of\ alternatives$$

and $x = \{x_{j,} j \in I_n\}$ is set of all parameters. Then FSS over v is a function defined by

$$H_i : X_i \to P(V)$$

where H_i a mapping is given by [22]

Step2. Construct decision matrix D for each decision makers.

Where $D = V_{i,j\in I_n} d_{ij}, d_{ij} = fx_i(x_j)$ is the criterion values of i^{th} alternatives received from the criterion, X_i is the parameter sets of decision makers D_p and $f_{x_i}$is the soft set which was constructed by D_p.

$$D = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix} \end{array} = \lfloor d_{ij} \rfloor_{m\times n}$$

Step3: Obtaining the WNDM of V. The WNDM is calculated as:

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} = \lfloor v_{ij} \rfloor_{m\times n}$$

where $v_{kt} = \sum_{i=1}^{n} f_{x_i}(x_k)(u_t), \ \forall i, k, t \in I_n$

Step4.

$$k(u_j) = \sum_{i=1}^{n} v_{ji}$$

where, k(u_j) is decision values of u_j.Thus the decision matrix of each alternative values for the deciders is expressed as

$$R = \{k(u_1), \dots \dots \dots k(u_n)\}$$

Step5. Ranking the preference order.

An Application

Step1. Defining the problem

Suppose that a car dealer has a set of various types of cars (universal set-alternatives)

$$v = \{v_1, v_2, v_3\}$$

which may be categorized by a set of all parameters

$$Y = \{y_1, y_2, y_3\} \text{For } j = 1, 2, 3.$$

The parameters Y_j stand for "luxuries", "automatic” and "manual" respectively.

Assume that three decision-makers come to the car dealer to buy a car.

Firstly, each decision-maker has their own choice about car and they consider their own set of parameters.

Then they can construct their fuzzy-soft-sets. Next, by using the fuzzy-soft-set and TOPSIS-decision making method we select a car on the basis of parameters of decision makers.

Step2. Construct decision matrix D for each decision- makers. We can construct fuzzy soft sets of decision-makers, D_i in a tabular form respectively as Fuzzy soft sets of decision-maker, D_1 is

$$\begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} & \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.2 & 1 & 0 \\ 0.4 & 0 & 0.3 \end{bmatrix} \end{array}$$

Fuzzy soft sets of decision-maker, D_2 is

$$\begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} & \begin{bmatrix} 0.2 & 0.5 & 0.4 \\ 0.3 & 0 & 2 \\ 0.1 & 0.5 & 0.4 \end{bmatrix} \end{array}$$

Fuzzy soft sets of decision-maker, D_3 is

$$\begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} & \begin{bmatrix} 0.4 & 0 & 0.3 \\ 0.5 & 0 & 0.1 \\ 0.4 & 0.2 & 0.1 \end{bmatrix} \end{array}$$

Step3. Creating the weighted-normalized-decision-matrix V.
Now compute the weights corresponding to each parameter.

$$v_{11=}\sum_{i=1}^{3} f_{y_{i(y1)}}\,(v_1) = f_{y_1(y1)(v_1)}+f_{y_2(y1)(v_1)}+\,f_{y_3(y1)(v_1)}$$

$$= 0.3 + 0.2 + 0.4 = 0.9$$

$$v_{12=}\sum_{i=1}^{3} f_{y_{i(y1)}}\,(v_2) = f_{y_1(y1)(v_2)}+f_{y_2(y1)(v_2)}+\,f_{y_3(y1)(v_2)}$$

$$= 0.2 + 0.3 + 0.5 = 01$$

$$v_{13=}\sum_{i=1}^{3} f_{y_{i(y1)}}\,(v_3) = f_{y_1(y1)(v_3)}+f_{y_2(y1)(v_3)}+\,f_{y_3(y1)(v_3)}$$

$$= 0.4 + 0.1 + 0.4 = 0.9$$

$$v_{21=}\sum_{i=1}^{3} f_{y_{i(y2)}}\,(v_1) = f_{y_1(y2)(v_1)} + f_{y_2(y2)(v_1)} + f_{y_3(y2)(v_1)}$$

$$= 0.5 + 0.5 + 0 = 01$$

$$v_{22=}\sum_{i=1}^{3} f_{y_{i(y2)}}\,(v_2) = f_{y_1(y2)(v_2)} + f_{y_2(y2)(v_2)} + f_{y_3(y2)(v_2)}$$

$$= 0.0 + 0.0 + 01 = 01$$

$$v_{23=}\sum_{i=1}^{3} f_{y_{i(y2)}}\,(v_3) = f_{y_1(y2)(v_3)} + f_{y_2(y2)(v_3)} + f_{y_3(y2)(v_3)}$$

$$= 0.0 + 0.5 + 0.2 = 0.7$$

$$v_{31=}\sum_{i=1}^{3} f_{y_{i(y3)}}\,(v_1) = f_{y_1(y3)(v_1)} + f_{y_2(y3)(v_1)} + f_{y_3(y3)(v_1)}$$

$$= 0.2 + 0.4 + 0.3 = 0.9$$

$$v_{32=}\sum_{i=1}^{3} f_{y_{i(y3)}}\,(v_2) = f_{y_1(y3)(v_2)} + f_{y_2(y3)(v_2)} + f_{y_3(y3)(v_2)}$$

$$= 0.0 + 0.2 + 0.1 = 0.3$$

$$v_{33=}\sum_{i=1}^{3} f_{y_{i(y3)}}\,(v_3) = f_{y_1(y3)(v_3)} + f_{y_2(y3)(v_3)} + f_{y_3(y3)(v_3)}$$

$$= 0.3 + 0.4 + 0.1 = 0.8$$

Then the weight matrix is obtained as

$$V = \begin{bmatrix} 0.9 & 1 & 0.9 \\ 0.1 & 1 & 0.7 \\ 0.9 & 0.3 & 0.8 \end{bmatrix}$$

Step4: Creating the decision matrix (vector), R.
Now, calculate the individual elements of the R matrix.

$$k(v_1) = \sum_{i=1}^{3} v_{1i} = v_{11} + v_{12} + v_{13} = 0.9 + 0.1 + 0.9 = 2.8$$

$$k(v_2) = \sum_{i=1}^{3} v_{1i} = v_{21} + v_{22} + v_{23} = 01 + 01 + 0.3 = 2.3$$

$$k(v_3) = \sum_{i=1}^{3} v_{1i} = v_{31} + v_{32} + v_{33} = 0.9 + 0.7 + 0.8 = 2.4$$

$$R = [2.8 \quad ,2.3 \quad ,2.4]$$

Step 5. Ranking the preference order.

Ranking of the alternatives would be created in the descending order of the valuesk (v_j) calculated in the fifth step. So when the fifth step in the calculation of the evaluation of the candidate cars (alternatives) from small to large $k\,(v_2) < k(v_3) < k(v_1)$, the order form is realized in the form of ranking $v_2 < v_3 < v_1$. In other words, the most suitable car appears to be v_1.

3. Conclusion

We proposed a new method for selection in this paper. After verifying the accumulations on different situations it can be observed that the new method is quite simple to use and significant for accumulation. Also, it has less no of calculations than the original TOPSIS. In this method we deal with the indeterminate and fuzzy or ambiguous values. We get same result through soft set on TOPSIS and with fuzzy soft set on TOPSIS.

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