

Algebra II

(Ring Theory)

Credit Hours: 3

Prerequisites: Algebra I

Specific Objectives of course: This is a course in advanced abstract algebra, which builds on the concepts learnt in Algebra I. The objectives of the course are to introduce students to the basic ideas and methods of modern algebra and enable them to understand the idea of a ring and an integral domain, and be aware of examples of these structures in mathematics; appreciate and be able to prove the basic results of ring theory; appreciate the significance of unique factorization in rings and integral domains.

Course Outline:

Rings: Definition, examples. Quadratic integer rings. Examples of non-commutative rings. The Hamilton quaternions. Polynomial rings. Matrix rings. Units, zero-divisors, nilpotents, idempotents. Subrings, Ideals. Maximal and prime Ideals. Left, right and two-sided ideals; maximal and prime ideals. Operations with ideals. The ideal generated by a set. Quotient rings. Ring homomorphism. The isomorphism theorems, applications. Finitely generated ideals. Rings of fractions.

Integral Domain: The Chinese remainder theorem. Divisibility in integral domains, greatest common divisor, least common multiple. Euclidean domains. The Euclidean algorithm. Principal ideal domains. Prime and irreducible elements in an integral domain. Gauss lemma, irreducibility criteria for polynomials. Unique factorization domains. Finite fields. Polynomials in several variables. Symmetric polynomials. The fundamental theorem of symmetric polynomials.

Recommended Books:

1. J. Rose, *A Course on Group Theory*, Cambridge University Press, 1978.
2. I. N. Herstein, *Topics in Algebra*, Xerox Publishing Company, 1964.
3. P. M. Cohn, *Algebra*, John Wiley and Sons, London, 1974.
4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, *Basic Abstract Algebra*, Cambridge University Press, 1986.
5. J. B. Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley Publishing Company, 2002.
7. Vivek Sahai and Vikas Bist, *Algebra*, Narosa Publishing House, 1999.
8. D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3rd Edition, Addison-Wesley Publishing Company, 2004.