**University of Management and Technology**

**School of Science**

***Department of Mathematics***

**Course Code:** MTH 725

**Course Title: Applications of fractional Calculus to heat and mass transfer problems**

**Program:PH.D**

**Prerequisites:** Calculus and Analytical Geometry includes limits, continuity, integration and differentiation. Ordinary differential equations, Partial differential equations, Vector analysis, Basic Physics includes mechanics & heat transfer

**Course Outline:**

Heat and Mass transfer: Basic derivation of transport properties based on kinetic theory of gases. Use of dimensionless parameters; Gr, Sh, Nu, Sc, Re, Pr. Basic heat transfer equations and mechanisms-steady state and unsteady state heat transfer Heat transfer coefficients. Derivation of energy equation, momentum equation and diffusion equation, Application of dimensional analysis to heat flow. Basic mass transfer equation. Mass transfer equations and models.Heat transfer by free convection and condensing vapors, boiling liquid, Radiation heat transfer between solids separated by a transparent (diathermal). Participating medium, Radiation heat transfer in absorbed medium, calculation of heat exchangers. Fick’s law, Fourier law of heat transfer, Newton’s law of cooling, Thermal radiation, mass diffusion, Convection, types of convection, MHD, Porous medium, Newtonian heating, first order chemical reaction, Soret effect.

**Fractional Calculus and its Application**:

This course is about applications of fractional calculus in the field of applied sciences and engineering like industrial engineering, mechanical engineering and chemical engineering as well. The last decades prove that derivatives and integrals of arbitrary order are very convenient for describing properties of real materials e.g. polymers. The new fractional order models are more satisfying than former integer-order ones. Fractional derivatives are an excellent tool for describing the memory and properties of viscous materials and processes while in integer order models such effects are neglected. The fractional calculus finds also applications in different fields of science including theory of fractals, numerical analysis, physics, engineering, biology, economics and finance. History of fractional Calculus, properties of fractional Calculus, basic definitions, differential and integral representation of fractional derivatives operators, types of fractional derivative operators, relation between fractional derivatives operators, advantage of derivative operators, generalized special functions like, Gamma function, Beta functions, relation between Gamma and Beta functions, Mittage Leffler functions, G-functions, Hartely function, generalized G-function, Fox-H function and their properties, modeling of some real world problems with fractional derivatives operators, Application of Laplace transform with fractional derivatives, Applications of Fourier transform with fractional derivatives, Solution of heat transfer problem in terms of Wright’ s functions in one dimension. We will solve some latest models of heat and mass transfer for Newtonian and non-Newtonian fluids, next we will extend these models to larger class of non-Newtonian fluids with three different approaches of fractional derivatives and draw some comparison between these models. At the end, some applications of fractional calculus in solving real life problems like Science including theory of fractals, numerical analysis, physics, engineering, biology, economics and finance etc. will be studied.

**Learning Outcomes:**

* Upon completion of this module, students should be able to:
* Demonstrate an understanding of heat and mass transfer modes and models
* Demonstrate an understanding of the different types of interface reactions
* Apply principles of heat and mass transfer phenomena to metallurgical processes
* Solve process and materials related problems using heat and mass transfer modes phenomena principles.

**Expected outcomes:**

After successfully completing the course, students should be

* Comfortable with applying fractional calculus technique in any classical model of real word problem.
* Able to comprehend above mentioned classes in fractional calculus
* Able to understand and apply the concepts of fractional derivatives approach of some classical models in the existence literature.
* Compare and critical analysis of fractional calculus in heat and mass transfer flow of Newtonian and non-Newtonian fluids.
* Able to read, understand and explore research article.
* Able to write at least term paper.

**Recommended Books:**

1. Daniel W. Mackowski (2010). Conduction Heat Transfer. Mechanical Engineering Department Auburn University 304p.
2. Heat Transfer note prepared for Metallurgical Engineering Students, University of Namibia
3. C.H.R. Friedrich, Relaxation and retardation functions of the Maxwell model with fractional derivatives, Computers and mathematics with application (1991) 151–158.
4. A. Gemant, On Fractional Differentials, Computers and Mathematics with Application, 1938, 540–549.
5. L.I. Palade, P. Attane, R.R. Huilgoland, B. Mena, Anomalous Stability Behavior of a Properly Invariant Constitutive Equation Which Generalizes Fractional Derivative Models, Computers and Mathematics With Application, 1999, 315–329.
6. Podlubny I (1999) Fractional differential equations. New York Academic Press, Cambridge
7. Bagley RL (1983) A theoretical basis for the application of fractional calculus to viscoelasticity. J Rheol 27:201–210.

**Related Recent Papers:**

1. Caputo M, Fabrizio M. A new definition of fractional derivative without 413 singular kernel. Progr Fract Differ Appl 2015; 1 (2):1–13. 414
2. Atangana A. On the new fractional derivative and application to 415 nonlinear Fisher’s reaction–diffusion equation. Appl Math Comput 416 2016; 273:948–56. 417
3. D. Vieru, C. Fetecau, C. Fetecau, Time fractional free convection flow near a vertical plate with Newtonian heating and mass diffusion, Int. J. Thermal Science. 19 (2015) 85–98.